EXAM V
CALCULUS AB
SECTION I PART A
Time-55 minutes
Number of questions-28

## A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:
(1) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.
(2) The inverse of a trigonometric function $f$ may be indicated using the inverse function notation $f^{-1}$ or with the prefix "arc" (e.g., $\sin ^{-1} x=\arcsin x$ ).

1. If $y=\cos ^{2}(2 x)$, then $\frac{d y}{d x}=$
(A) $2 \cos 2 x \sin 2 x$
(B) $-4 \sin 2 x \cos 2 x$
(C) $2 \cos 2 x$
(D) $-2 \cos 2 x$
(E) $4 \cos 2 x$
2. A slope field for a differential equation $\frac{d y}{d x}=f(x, y)$ is given in the figure at the right. Which of the following statements are true?
I. The value of $\frac{d y}{d x}$ at the point $(2,2)$ is approximately 1 .
II. As $y$ approaches 8 the rate of change of $y$ approaches zero.
III. All solution curves for the differential equation have the same slope for a given value of $x$.

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, III

Ans

3. The slope of the line tangent to the graph of $y=\ln \sqrt{x}$ at $\left(e^{2}, 1\right)$ is
(A) $\frac{e^{2}}{2}$
(B) $\frac{2}{e^{2}}$
(C) $\frac{1}{2 e^{2}}$
(D) $\frac{1}{2 e}$
(E) $\frac{1}{e}$

## Ans

4. Which of the following functions is both continuous and differentiable at all $x$ in the interval $-2 \leq x \leq 2$ ?
(A) $f(x)=\left|x^{2}-1\right|$
(B) $f(x)=\sqrt{x^{2}-1}$
(C) $f(x)=\sqrt{x^{2}+1}$
(D) $f(x)=\frac{1}{x^{2}-1}$
(E) none of these
5. Find the point on the graph of $y=\sqrt{x}$ between $(1,1)$ and $(9,3)$ at which the tangent to the graph has the same slope as the line through $(1,1)$ and $(9,3)$.
(A) $(1,1)$
(B) $(2, \sqrt{2})$
(C) $(3, \sqrt{3})$
(D) $(4,2)$
(E) none of the above

## Ans

$\square$
6. Consider the function $f(x)=\frac{x^{4}}{2}-\frac{x^{5}}{10}$. The derivative of $f$ attains its maximum value at $x=$
(A) 3
(B) 4
(C) 5
(D) 0
(E) there is no maximum

Ans
7. The acceleration, $a(t)$, of a body moving in a straight line is given in terms of time $t$ by $a(t)=4-6 t$. If the velocity of the body is 20 at $t=0$ and if $s(t)$ is the distance of the body from the origin at time $t$, what is $s(3)-s(1)$ ?
(A) -10
(B) 0
(C) 10
(D) 20
(E) 30
8. $\lim _{x \rightarrow 1}\left(\frac{\sqrt{x+3}-2}{1-x}\right)$
(A) 0.5
(B) 0.25
(C) 0
(D) -0.25
(E) -0.5

Ans
9. Let $f$ be defined by $f(x)=\left\{\begin{array}{lr}\frac{x^{2}-2 x+1}{x-1} \text { for } x \neq 1 \\ k \quad \text { for } x=1 .\end{array}\right.$

Determine the value of $k$ for which $f$ is continuous for all real $x$.
(A) 0
(B) 1
(C) 2
(D) 3
(E) none of the above
10. The average value of $f(x)=e^{2 x}+1$ on the interval $0 \leq x \leq \frac{1}{2}$ is
(A) $e$
(B) $\frac{e}{2}$
(C) $\frac{e}{4}$
(D) $2 e-1$
(E) $\frac{e^{2 x}+1}{2}$
11. A point moves on the $x$-axis in such a way that its velocity at time $t>0$ is given by $v=\frac{e^{t}}{t}$. At what value of $t$ does $v$ attain its minimum?
(A) 0
(B) 1
(C) $e$
(D) -1
(E) There is no minimum value of $v$.

12. $\int \frac{4 x}{1+x^{2}} d x=$
(A) $4 \operatorname{Arctan} x+C$
(B) $\frac{4}{x} \operatorname{Arctan} x+C$
(C) $\frac{1}{2} \ln \left(1+x^{2}\right)+C$
(D) $2 \ln \left(1+x^{2}\right)+C$
(E) $2 x^{2}+4 \ln |x|+C$

Ans
$\square$
13. Let $f(x)=x^{4}+a x^{2}+b$. The graph of $f$ has a relative maximum at $(0,1)$ and an inflection point when $x=1$. The values of $a$ and $b$ are
(A) $a=1, \quad b=-6$
(B) $a=1, \quad b=6$
(C) $a=-6, \quad b=5$
(D) $a=-6, \quad b=1$
(E) $a=6, \quad b=1$
14. $\int_{1}^{2} \frac{x^{2}-x}{x^{3}} d x=$
(A) $\ln 2-\frac{1}{2}$
(B) $\ln 2+\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 0
(E) $\frac{1}{4}$

15. The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube?
(A) $5 \mathrm{in}^{3} / \mathrm{sec}$
(B) $10 \mathrm{in}^{3} / \mathrm{sec}$
(C) $15 \mathrm{in}^{3} / \mathrm{sec}$
(D) $20 \mathrm{in}^{3} / \mathrm{sec}$
(E) $25 \mathrm{in}^{3} / \mathrm{sec}$
16. $\int_{0}^{\sqrt{3}} \frac{x d x}{\sqrt{1+x^{2}}}=$
(A) $\frac{1}{2}$
(B) 1
(C) 2
(D) $\ln 2$
(E) $\operatorname{Arctan} 2-\frac{\pi}{4}$
17. Which of the following is true about the graph of $f(x)=\ln \left|x^{2}-4\right|$ in the interval $(-2,2)$ ?
(A) $f$ is increasing.
(B) $f$ attains a relative minimum at $(0,0)$.
(C) $f$ has a range of all real numbers.
(D) $f$ is concave down.
(E) $f$ has an asymptote at $x=0$.
18. If $g(x)=\operatorname{Arcsin} 2 x$, then $g^{\prime}(x)=$
(A) $2 \operatorname{Arccos} 2 x$
(B) $2 \csc 2 x \cot 2 x$
(C) $\frac{2}{1+4 x^{2}}$
(D) $\frac{2}{\sqrt{4 x^{2}-1}}$
(E) $\frac{2}{\sqrt{1-4 x^{2}}}$
19. $\int x\left(x^{2}-1\right)^{4} d x=$
(A) $\frac{1}{10}\left(x^{2}\right)\left(x^{2}-1\right)^{5}+C$
(B) $\frac{1}{10}\left(x^{2}-1\right)^{5}+C$
(C) $\frac{1}{5}\left(x^{3}-x\right)^{5}+C$
(D) $\frac{1}{5}\left(x^{2}-1\right)^{5}+C$
(E) $\frac{1}{5}\left(x^{2}-x\right)^{5}+C$
20. If $y=e^{k x}$, then $\frac{d^{5} y}{d x^{5}}=$
(A) $k^{5} e^{x}$
(B) $k^{5} e^{k x}$
(C) $5!e^{k x}$
(D) $5!e^{x}$
(E) $5 e^{k x}$
21. The graph of $f$ is shown at the right. Which of the following statements are true?
I. $f(2)>f^{\prime}(1)$
II. $\int_{0}^{1} f(x) d x>f^{\prime}(3.5)$
III. $\int_{-1}^{1} f(x) d x>\int_{-1}^{2} f(x) d x$
(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, III

Ans
22. If $g(x)=\sqrt{x}(x-1)^{2 / 3}$, then the domain of $g^{\prime}$ is
(A) $\{x \mid 0<x\}$
(B) $\{x \mid x \neq 0$ and $x \neq 1\}$
(C) $\{x \mid 0<x<1$ or $x>1\}$
(D) $\{x \mid 0<x<1\}$
(E) $\{x \mid$ all real numbers $\}$

Ans
23. A point moves on the $x$-axis so that its distance from the origin at time $t$ is given by $10 t-4 t^{2}$. What is the total distance covered by the point between $t=1$ and $t=2$ ?
(A) 1.0
(B) 1.5
(C) 2.0
(D) 2.5
(E) 3.0
$\square$
24. At which point on the graph of $y=g(x)$ below is $g^{\prime}(x)=0$ and $g^{\prime \prime}(x)=0$ ?

(A) A
(B) B
(C) C
(D) D
(E) E

Ans

25. If $y$ is a differentiable function of $x$, then the slope of the tangent to the curve $x y-2 y+4 y^{2}=6$ at the point where $y=1$ is
(A) $\frac{1}{12}$
(B) $-\frac{1}{10}$
(C) $-\frac{1}{6}$
(D) $\frac{1}{4}$
(E) $-\frac{5}{6}$

Ans
26. The area of the region bounded above by $y=1+\sec ^{2} x$, below by $y=0$, on the left by $x=0$ and on the right by $x=\frac{\pi}{4}$ is approximately
(A) 1
(B) 1.25
(C) 1.5
(D) 1.75
(E) 2

Ans
27. A solution of the equation $\frac{d y}{d x}+2 x y=0$ that contains the point $(0, e)$ is
(A) $y=e^{1-x^{2}}$
(B) $y=e^{1+x^{2}}$
(C) $y=e^{1-x}$
(D) $y=e^{1+x}$
(E) $y=e^{x^{2}}$
28. Which of the following are true about the function $F(x)=\int_{1}^{x} \ln (2 t-1) d t$ ?
I. $F(1)=0$
II. $F^{\prime}(1)=0$
$F^{\prime \prime}(1)=1$
(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II, III
(E) none

## EXAM V

CALCULUS AB
SECTION I PART B
Time-50 minutes
Number of questions- $\mathbf{1 7}$

## A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:
(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
(2) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.
(3) The inverse of a trigonometric function $f$ may be indicated using the inverse function notation $f^{-1}$ or with the prefix "arc" (e.g., $\sin ^{-1} x=\arcsin x$ ).

1. How many points of inflection does the graph of $y=\cos x+\frac{1}{3} \cos 3 x-\frac{1}{5} \cos 5 x$ have on the interval $0 \leq x \leq \pi$ ?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5
2. Oil is leaking from a tanker at the rate of $R(t)=500 e^{-0.2 t}$ gallons per hour, where $t$ is measured in hours. The amount of oil that has leaked out after 10 hours is closest to
(A) 2140 gals
(B) 2150 gals
(C) 2160 gals
(D) 2170 gals
(E) 2180 gals
3. The sale of lumber $S$ (in millions of square feet) for the years 1980 to 1990 is modeled by the function

$$
S(t)=0.46 \cos (0.45 t+3.15)+3.4
$$

where $t$ is the time in years with $t=0$ corresponding to the beginning of 1980. Determine the year when lumber sales were increasing at the greatest rate.
(A) 1982
(B) 1983
(C) 1984
(D) 1985
(E) 1986
4. The graph of $f$ over the interval $[1,9]$ is shown in the figure. Using the data in the figure, find a midpoint approximation with 4 equal subdivisions for $\int_{1}^{9} f(x) d x$.

(A) 20
(B) 21
(C) 22
(D) 23
(E) 24

Ans

5. Let the base of a solid be the first quadrant region enclosed by the $x$-axis and one arch of the graph of $y=\sin x$. If all cross sections perpendicular to the $x$-axis are squares, then the volume of the solid is approximately
(A) 0.52
(B) 0.79
(C) 1.05
(D) 1.57
(E) 2.00

6. If $f(x)=2 x+\sin x$ and the function $g$ is the inverse of $f$, then $g^{\prime}(2)=$
(A) 0.32
(B) 0.34
(C) 0.36
(D) 0.38
(E) 0.40
7. Administrators at Massachusetts General Hospital believe that the hospital's expenditures $E(B)$, measured in dollars, are a function of how many beds $B$ are in use with

$$
E(B)=14000+(B+1)^{2}
$$

On the other hand, the number of beds $B$ is a function of time $t$, measured in days, and it is estimated that

$$
B(t)=20 \sin \left(\frac{t}{10}\right)+50
$$

At what rate are the expenditures decreasing when $t=100$ ?
(A) 120 dollars/day
(B) 125 dollars/day
(C) 130 dollars/day
(D) 135 dollars/day
(E) 140 dollars/day
8. Let $f$ be a function that has domain $[-2,5]$. The graph of $f^{\prime}$ is shown at the right. Which of the following statements are TRUE?
I. $f$ has a relative maximum at $x=-1$.
II. $f$ has an absolute minimum at $x=0$.
III. The graph of $f$ is concave down for $-2<x<0$.
IV. The graph of $f$ has inflection points at $x=0$ and $x=2$ and $x=3$.
(A) I, II, IV
(B) I, III, IV
(C) II, III, IV
(D) I, II, III
(E) I, II, III, IV

Ans

9. On which interval is the graph of $f(x)=4 x^{3 / 2}-3 x^{2}$ both concave down and increasing?
(A) $(0,1)$
(B) $\left(0, \frac{1}{2}\right)$
(C) $\left(0, \frac{1}{4}\right)$
(D) $\left(\frac{1}{4}, \frac{5}{4}\right)$
(E) $\left(\frac{1}{4}, 1\right)$
10. The average rate of change of the function $f(x)=x^{2}-\frac{1}{e^{x}}$ over the interval $[0,3]$ equals the instantanous rate of change of $f$ at $x=$
(A) 0.313
(B) 1.553
(C) 2.573
(D) 3.317
(E) 9.950

11. If $\sin 3 x-1=\int_{a}^{x} f(t) d t$, then the value of $a$ is
(A) 0
(B) 1
(C) -1
(D) $\frac{\pi}{3}$
(E) $\frac{\pi}{6}$

Ans
12. If $x y^{2}=20$ and $x$ is decreasing at the rate of 3 units per second, the rate at which $y$ is changing when $y=2$ is nearest to
(A) -0.6 units $/ \mathrm{sec}$
(B) -0.2 units/sec
(C) 0.2 units/sec
(D) 0.6 units/sec
(E) 1.0 units $/ \mathrm{sec}$
13. An approximation for $\int_{-1}^{2} e^{\sin (1.5 x-1)} d x$ using a right-hand Riemann sum with three equal subdivisions is nearest to
(A) 2.5
(B) 3.5
(C) 4.5
(D) 5.5
(E) 6.5
14. If $f(x)$ is defined on $-\pi \leq x \leq \pi$ and $\frac{d y}{d x}=\frac{\cos x}{x^{2}+1}$, which of the following statements about the graph of $y=f(x)$ is true?
(A) The graph has no relative extremum.
(B) The graph has one point of inflection and two relative extrema.
(C) The graph has two points of inflection and one relative extremum.
(D) The graph has two points of inflection and two relative extrema.
(E) The graph has three points of inflection and two relative extrema.
15. The graph of the function $f$ is shown at the right. If the function $G$ is defined by $G(x)=\int_{-4}^{x} f(t) d t$, for $-4 \leq x \leq 4$, which of the following statements about $G$ are true?
I. $G$ is increasing on $(1,2)$.
II. $G$ is decreasing on $(-4,-3)$.

III. $G(0)<0$.
(A) None
(B) II only
(C) III only
(D) II and III only
(E) I and II only
16. The function $f$ is defined on all the reals such that $f(x)=\left\{\begin{array}{l}x^{2}+k x-3 \text { for } x \leq 1 \\ 3 x+b \text { for } x>1 .\end{array}\right.$

For which of the following values of $k$ and $b$ will the function $f$ be both continuous and differentiable on its entire domain?
(A) $k=-1, b=-3$
(B) $k=1, b=3$
(C) $k=1, b=4$
(D) $k=1, b=-4$
(E) $k=-1, b=6$.
17. A particle moves along the $x$-axis with velocity at time $t$ given by: $v(t)=t+2 \sin t$. If the particle is at the origin when $t=0$, its position at the time when $v=6$ is $x=$
(A) 17.14
(B) 19.16
(C) 23.18
(D) 29.20
(E) 39.30

## A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.
- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.
- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in mathematical notation rather than calculator syntax. For example, 5
$\int_{1} x^{2} d x$ may not be written as $\operatorname{fn} \operatorname{Int}\left(X^{2}, X, 1,5\right)$.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. The position of a particle moving on the $x$-axis at time $t>0$ seconds is: $x(t)=e^{t}-\sqrt{t}$ feet.
(a) Find the average velocity of the particle over the interval $1 \leq t \leq 3$.
(b) In what direction and how fast is the particle moving at $t=1$ seconds?
(c) For what values of $t$ is the particle moving to the right?
(d) Find the position of the particle when its velocity is zero.
2. Water flowed into a tank at an increasing rate $r(t)$ from $t=0$ to $t=5$ minutes. The rate of flow, $r(t)$, in cubic meters per minute ( $\mathrm{m}^{3} / \mathrm{min}$ ), was measured at one minute intervals with the result shown in the table below.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r(t)$ | 4 | 5 | 7 | 11 | 12 | 14 |

(a) Give the best upper and lower estimates for the total amount of water that flowed into the tank for $0 \leq t \leq 5$. Indicate units of measure.
(b) Suppose you use the average of the upper and lower estimates found in part (a) as your approximation for the total amount of water that flowed into the tank, what is the maximum error for this approximation?
(c) You are now informed that for $1 \leq t \leq 3$ the rate of flow was exactly $r(t)=t^{2}-t+5$ $\mathrm{m}^{3} / \mathrm{min}$. What is the exact amount of water that flowed into the tank from $t=1$ to $t=3$ ?
3. Let $R$ be the first quadrant region enclosed by the graph of $y=2 e^{-x}$ and the line $x=k$.
(a) Find the area of $R$ in terms of $k$.
(b) Find the volume of the solid generated when $R$ is rotated about the $x$-axis in terms of $k$.
(c) What is the volume in part (b) as $k \rightarrow \infty$ ?

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION. DURING THE TIMED PORTION FOR PART B, YOU MAY GO BACK AND CONTINUE TO WORK ON THE PROBLEMS IN PART A WITHOUT THE USE OF A CALCULATOR.
4. Consider the differential equation $\frac{d y}{d x}=\frac{x y}{\left(x^{2}+4\right)}$.
(a) On the axes provided, sketch a slope field for the given differential equation at the fourteen points indicated.
(b) Sketch the solution curve that contains the point $(-2,2)$.
(c) Find a general solution to the differential equation.
(d) Find the particular solution to the differential equation that satisfies the initial condition $y(0)=4$.

5. Graphs of functions $f$ and $g$ are shown below.


Let $h(x)=f[g(x)]$. Use the graphs to answer the following questions about the function $h$.
(a) Approximate the critical points of $h$.
(b) Where does $h$ have a local minimum?
(c) Where is the function $h$ decreasing?
(d) Sketch a graph of $h$ on the axes provided.
6. Let $f$ be the function defined by $f(x)=\ln \left(\frac{x}{x+1}\right)$.
(a) What is the domain of $f$ ?
(b) Find $f^{\prime}(x)$.
(c) Find an equation for the tangent line to the graph of $f$ at the point $(1, f(1))$.
(d) Write an expression for $g^{\prime}(x)$, where $g$ is the inverse function of $f$.

